

Attendance Problems.

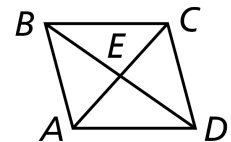
1. Find AB for A(-3, 5) and B(1, 2).

2. Find the slope of \overline{JK} for J(-4, 4) and K(3, -3).

 $ABCD$ is a parallelogram.

3. Why is $\angle ABC \cong \angle CDA$?

4. Why is $\angle AEB \cong \angle CED$?

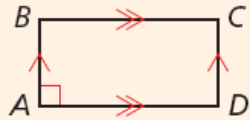
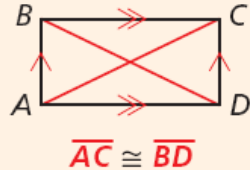


I can prove that a given quadrilateral is a rectangle, rhombus, or square.

Common Core: CC.9-12.G.CO.11 Prove theorems about parallelograms.



Theorems **Conditions for Rectangles**

THEOREM	EXAMPLE
6-5-1 If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle. (\square with one rt. $\angle \rightarrow$ rect.)	
6-5-2 If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (\square with diags. $\cong \rightarrow$ rect.)	

Refer to video example 1.

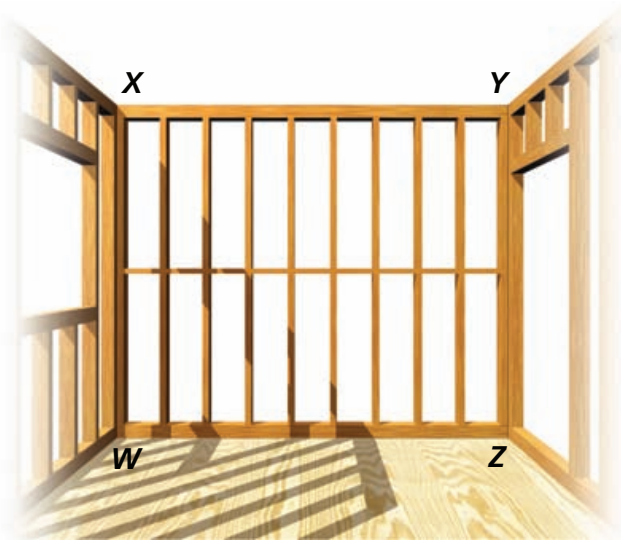
A carpenter built a wood frame for a window so that $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$, and $\overline{AD} \cong \overline{BC}$. Why must the frame be a rectangle?



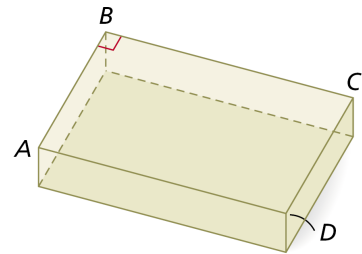
1 **Carpentry Application**

A contractor built a wood frame for the side of a house so that $\overline{XY} \cong \overline{WZ}$ and $\overline{XW} \cong \overline{YZ}$. Using a tape measure, the contractor found that $XZ = WY$. Why must the frame be a rectangle?

Both pairs of opposite sides of $WXYZ$ are congruent, so $WXYZ$ is a parallelogram. Since $XZ = WY$, the diagonals of $\square WXYZ$ are congruent. Therefore the frame is a rectangle by Theorem 6-5-2.



Example 1. A manufacturer builds a mold from a desktop so that $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$, & $m\angle ABC = 90^\circ$. Why must $ABCD$ be a rectangle?



5. Guided Practice. A carpenter's square can be used to test that an angle is a right angle. How could the contractor use a carpenter's square to check that the frame is a rectangle?



Theorems Conditions for Rhombuses

THEOREM	EXAMPLE
6-5-3 If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. $(\square \text{ with one pair cons. sides } \cong \rightarrow \text{rhombus})$	
6-5-4 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. $(\square \text{ with diags. } \perp \rightarrow \text{rhombus})$	
6-5-5 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. $(\square \text{ with diag. bisecting opp. } \angle \rightarrow \text{rhombus})$	

Caution

In order to apply Theorems 6-5-1 through 6-5-5, the quadrilateral must be a parallelogram.

Remember!

You can also prove that a given quadrilateral is a rectangle, rhombus, or square by using the definitions of the special quadrilaterals.

Parent: Your geometry teacher thinks you need extra homework.

Student: Tell her I want proof!

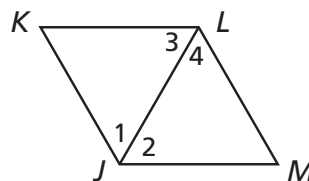
"We dedicated ourselves to a powerful idea -- organic law rather than naked power. There seems to be universal acceptance of that idea in the nation." -- *Supreme Court Justice Potter Steart*

Theorem 6-5-5

Given: $JKLM$ is a parallelogram.
 \overline{JL} bisects $\angle KJM$ and $\angle KLM$.

Prove: $JKLM$ is a rhombus.

Proof:



Statements	Reasons
1. $JKLM$ is a parallelogram. \overline{JL} bisects $\angle KJM$ and $\angle KLM$.	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Def. of \angle bisector
3. $\overline{JL} \cong \overline{JL}$	3. Reflex. Prop. of \cong
4. $\triangle JKL \cong \triangle JML$	4. ASA Steps 2, 3
5. $\overline{JK} \cong \overline{JM}$	5. CPCTC
6. $JKLM$ is a rhombus.	6. \square with one pair cons. sides $\cong \rightarrow$ rhombus

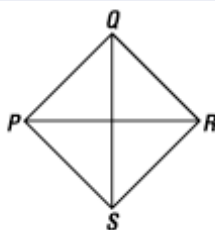
Refer to video example 2.

A.

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

Given:
 $\overline{PQ} \cong \overline{SR}$, $\overline{PS} \cong \overline{QR}$, $\overline{PQ} \perp \overline{QR}$,
and $\overline{PR} \perp \overline{QS}$.

Conclusion:
 $PQRS$ is a square.

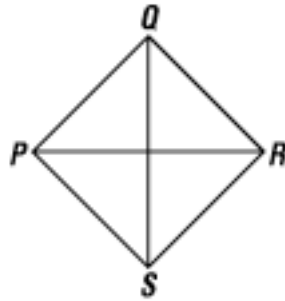


B.

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

Given:
 $\overline{PR} \perp \overline{QS}$

Conclusion:
 $PQRS$ is a rhombus.



2**Applying Conditions for Special Parallelograms**

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

A Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$,
 $\overline{AD} \perp \overline{DC}$, $\overline{AC} \perp \overline{BD}$

Conclusion: $ABCD$ is a square.

Step 1 Determine if $ABCD$ is a parallelogram.

$\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$ *Given*

$ABCD$ is a parallelogram. *Quad. with opp. sides $\cong \rightarrow \square$*

Step 2 Determine if $ABCD$ is a rectangle.

$\overline{AD} \perp \overline{DC}$, so $\angle ADC$ is a right angle. *Def. of \perp*

$ABCD$ is a rectangle. *\square with one rt. $\angle \rightarrow$ rect.*

Step 3 Determine if $ABCD$ is a rhombus.

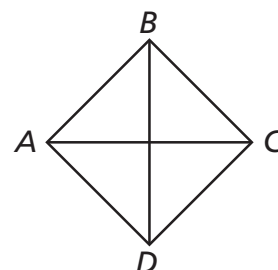
$\overline{AC} \perp \overline{BD}$ *Given*

$ABCD$ is a rhombus. *\square with diags. $\perp \rightarrow$ rhombus*

Step 4 Determine if $ABCD$ is a square.

Since $ABCD$ is a rectangle and a rhombus, it has four right angles and four congruent sides. So $ABCD$ is a square by definition.

The conclusion is valid.



B Given: $\overline{AB} \cong \overline{BC}$

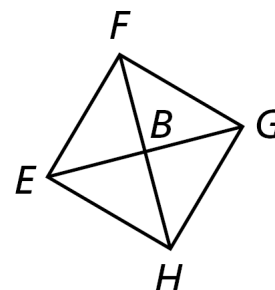
Conclusion: $ABCD$ is a rhombus.

The conclusion is not valid. By Theorem 6-5-3, if one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. To apply this theorem, you must first know that $ABCD$ is a parallelogram.

Example 2. Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

A. Given: $\overline{EF} \cong \overline{FG}$ & $\overline{EG} \perp \overline{FH}$

Conclusion: EFGH is rhombus.



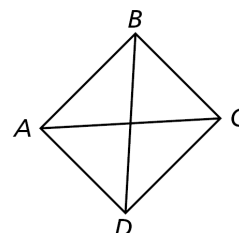
B. Given: $\overline{EB} \cong \overline{BG}$, $\overline{FB} \cong \overline{BH}$, $\overline{EG} \cong \overline{FH}$, $\triangle EBF \cong \triangle EBH$

Conclusion: EFGH is a square.

6. Guided Practice. Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

Given: $\angle ABC$ is a right angle.

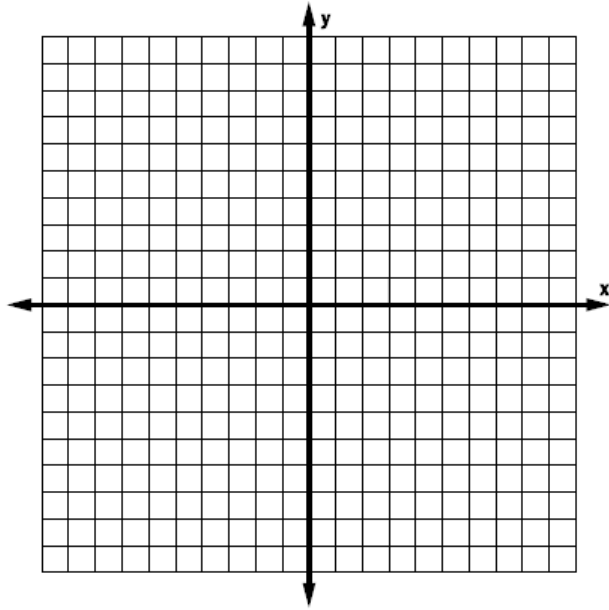
Prove: ABCD is a rectangle.



Refer to video example 3.

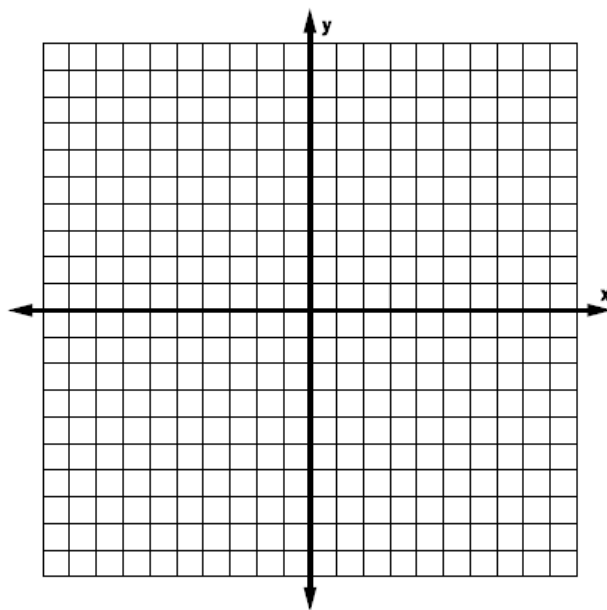
Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

$A(0, 2)$, $B(2, 7)$, $C(7, 5)$, $D(5, 0)$

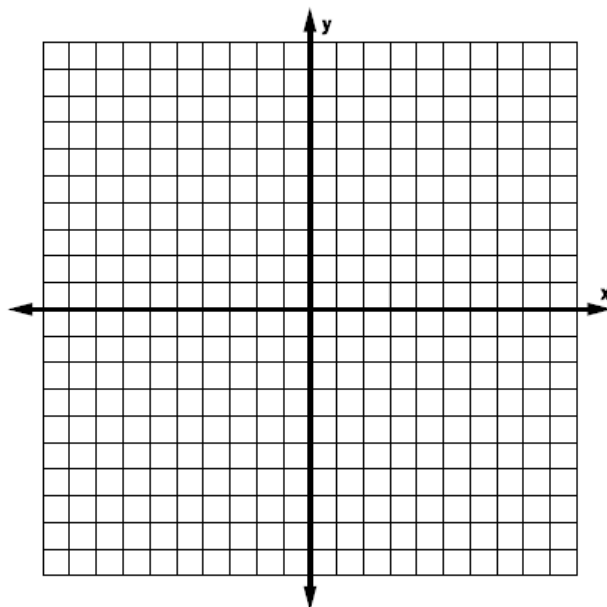


Guided Practice. Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

7. $K(-5, -1)$, $L(-2, 4)$, $M(3, 1)$, $N(0, -4)$



8. $P(-4, 6)$, $Q(2, 5)$, $R(3, -1)$, $S(-3, 0)$



6-5 Conditions for Special Parallelograms (p 434) 6-10, 14, 16, 17, 27, 29, 35.